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Heat Transfer in MHD Flow over A Stretching Sheet with Velocity and Thermal Slip Condition

M.C.Kemparaju[1] M. Subhas Abel[2] Mahantesh M.Nandeppanavar[3]

1.Department of Mathematics, Jyothy institute of Technology, Bangalore-560082, Karnataka, India

2.Department of Mathematics, Gulbarga University Gulbarga, Karnataka, India

3.Department of UG and PG Studies and Research in Mathematics, Government College, Gulbarga-585105, Karnataka, India

Abstract

The present work is concerned with the effects of surface slip conditions and thermal on an electrically conducting fluid over a non-isothermal stretching surface in the presence of a uniform transverse magnetic field. Similarity transformation is used to transform the partial differential equations describing the problem into a system of nonlinear ordinary differential equations which is solved analytically. The effects of various parameters on the velocity and temperature profiles as well as on the local skin-friction and the local Nusselt number are discussed in detail and displayed through graphs.

Keywords: MHD; Heat transfer; Slip conditions, Kumer's function, Similarity transformation

1. Introduction

The problem of flow and heat transfer in the boundary layer induced by a stretching surface in an otherwise ambient fluid is important in many industrial applications, such as the extrusion of plastic sheets, electronic chips, glass blowing, continuous casting and spinning of fibers.Crane [1] is the first to investigate analytically the problem of boundary layer flow of an incompressible viscous fluid over a linearly stretching surface. This problem was then extended by many authors. Gupta and Gupta[2] studied the effect of suction/blowing on heat and mass transfer over a stretching surface. Grubka and Bobba[3] analyzed heat transfer characteristics of a continuous stretching surface with variable temperature. Dutta et al.[4] studied the temperature field in the flow over a stretching sheet with uniform heat flux. Cortell[5] studied the effects of heat generation/absorption and suction/blowing on the flow and heat transfer of a fluid through a porous medium over a stretching surface.

In the above investigations. The authors dealt with hydrodynamic flow and heat transfer. Due to the importance of hydro magnetic flow and heat transfer problems in many engineering and industrial applications such as, MHD power generators, polymer processes and electro-chemistry it have attracted the attention of several authors. In many metallurgical processes, such as drawing, annealing and tinning of copper wires, involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid and that in the process of drawing, these strips are sometimes stretched. In all these cases the properties of the final product depend to great extent on the rate of cooling. By drawing such strips in an electrically conducting fluid subject to a magnetic field., the rate of cooling can be controlled and a final product of desired characteristics can be achieved. Pavlov [6] studied the effect of the magnetic field on the flow of an electrically conducting fluid on a stretching surface. Chakarbarti and Gupta [7] extended Pavlov's work to study the heat transfer when a uniform suction is applied at the stretching surface .Anderson[8] analytically studied the flow of an electrically conducting fluid on a linearly stretching surface with a magnetic field. Vajravelu and Rollins [9] analyzed heat transfer characteristics in an electrically conducting fluid over a stretching sheet with either a prescribed temperature or a prescribed heat flux in the presence of internal heat generation or absorption and a transfer magnetic field. Char [10] obtained exact solutions for the heat transfer in an electrically conducting fluid past a stretching sheet subjected to a thermal boundary with either a prescribed temperature or a prescribed heat flux in the presence of transverse magnetic field. Liu [11] studied the momentum, heat and mass transfer of hydro magnetic fluid past a stretching sheet in the presence of a uniform transverse magnetic field.

In all the above studies no-slip conditions are used. The no-slip condition is inadequate for viscous fluid. In rough and coated surface, the slip condition is used [12]. Wang [13] analyzed the entrained flow due to a stretching surface with partial slip. Anderson [14] studied the slip-flow of a Newtonian fluid past a linearly stretching sheet. Fang and Lee [15] investigated the boundary layer flow of a slightly rarefied gas free stream over a moving flat plate. Sajid [16] investigated the slip effects on the planar and axisymmetric flows in a viscous fluid. Fang et al. [17] studied the magneto hydrodynamic flow under slip condition over a permeable stretching surface. Wang[18] studied the viscous flow due to a stretching sheet with slip and suction. Mahmoud [19] studied the effect of slip conditions on the boundary layer flow and heat transfer of a slightly rarefied gas over a stretching surface in the presence of suction/blowing. Abbas et al. [20] studied the heat transfer of a viscous fluid over an oscillatory stretching sheet with slip condition. The magneto hydrodynamic flow and heat transfer characteristics for the boundary layer flow over a permeable stretching sheet in the presence of velocity and thermal slip conditions investigated by Hayath et al. [21].

The radiation effects are neglected in the above studies In the processes involving high temperatures such as glass production, polymer processes and furnace design and space technology applications such as gas cooled nuclear reactors, gas turbines, propulsion system, rocket combustion chamber and plasma physics, the radiation effects play an important role in such cases and can not be neglected. As a result, many studies have been carried out on the influences of thermal radiation on the heat transfer characteristics in different situations [22-26]. The aim of the present analysis is to study the heat transfer characteristic from a linearly stretching surface with power-law surface temperature in quiescent fluid in the presence of internal heat source , a uniform transverse magnetic field and slip conditions. Exact solution to the energy equation in terms of Kummer's functions is then obtained.

2.Formulation of the problem

Consider a steady, two-dimensional laminar incompressible flow of an electrically conducting fluid over a stretching surface issuing from a thin slit at the origin. The surface is stretched in its own plane with a velocity proportional to its distance from the slit. A uniform magnetic field of strength B_0 is applied in the $y-$direction which is normal to the flow direction. The magnetic Reynolds number is assumed to be small so that the induced magnetic field is neglected. It is also assumed that the applied electric field and the Hall effects are neglected. In addition, the influences of slip conditions and internal heat source are considered.

Under the above assumptions and boundary-layer approximations, the governing equations describing the problem are;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0{}^2}{\rho}u, \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\kappa\left(\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial x^2}\right)+Q(T-T_\infty) \tag{3}$$

Where u and v are the velocity components along the x and y directions respectively, ρ is the fluid density, T is the temperature of the fluid, μ is the viscosity of the fluid, κ is the thermal conductivity, c_p is the specific heat at constant pressure and σ is the electrical conductivity.

In the present study, we aim to investigate the thermal transport phenomenon for two general boundary thermal conditions, namely (i) constant surface temperature (CST) and (ii) prescribed surface temperature (PST) correspondingly, we consider the following similarity transformations and boundary conditions for considered flow as:

$$\left. u=cx\,f'(\eta),\qquad v=-\sqrt{\upsilon c}\,f(\eta)\quad and \qquad \eta=\sqrt{\frac{c}{\upsilon}}\,y \right\} \tag{4}$$

$$\left.\begin{array}{l} y=0:\ u=cx+\lambda_1\dfrac{\partial u}{\partial y},\ v=0,\ T=T_w+S_1\dfrac{\partial T}{\partial y}\,for(CST),T=A(\dfrac{x}{l})\theta(\eta)+S_2\dfrac{\partial T}{\partial y}\,for(PST) \\ y\to\infty;\qquad u\to\infty \end{array}\right\} \tag{5}$$

Using equations (4) and (5) in (2), we obtain

$$f''' + ff'' - f'^2 - Mf' = 0$$

The corresponding boundary conditions are;

$$\left.\begin{array}{lll} \eta=0: & f=0, & f'=1+\alpha_1 f'' \\ \eta\to\infty: & f'\to 0 & \end{array}\right\} \tag{8}$$

The momentum equation (6) with boundary conditions (8) has an exact solution in the form:

$$f(\eta)=B(1-e^{-\alpha\eta}) \tag{9}$$

Where $B = \dfrac{\alpha^2 - M}{\alpha}$, (10)

α is the real positive root of the cubic algebraic equation;

$$\alpha_1 \alpha^3 + \alpha^2 - \alpha_1 M \alpha - (1+M) = 0 \tag{11}$$

Where $\alpha_1 = \lambda_1 \sqrt{\dfrac{c}{\upsilon}}$ is the slip velocity parameter.

The physical quantities of interest are the local skin-friction coefficient C_{fx} which is defined as

$$C_{fx} = \frac{-2\tau_w}{\rho (cx)^2} \tag{12}$$

Where the surface shear stress τ_w is defined by

$$\tau_w = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0.} \tag{13}$$

Using the similarity variables, we get:

$$\frac{1}{2} C_{fx} \operatorname{Re}_x^{\frac{1}{2}} = -f^{"}(0) \tag{14}$$

Where $\operatorname{Re}_x = (\dfrac{cx^2}{\upsilon})$ is the local Reynolds Number.

3. Solution of heat transfer phenomena

We consider two general heating conditions, namely (1) CST and (2) PST

Case1: Constant surface temperature (CST)

Here we define the non dimensional temperature and the CST condition respectively as

$$\left.\begin{aligned} g(\eta) &= \frac{(T - T_\infty)}{(T_w - T_\infty)} \\ T &= T_\infty + (T_w - T_\infty)\, g(\eta), \end{aligned}\right\} \tag{15}$$

We obtain the energy equation and boundary conditions as

$$g^{"}(\eta) + prB(1 - e^{B\eta}) g^{'}(\eta) + H^{*} pr\, g(\eta) = 0 \tag{16}$$

$$g(0) = 1 + \alpha_2 g^{'}(0), \qquad g(\infty) = 0 \tag{17}$$

Where $pr = \dfrac{\mu c_p}{k}$, $H^* = \dfrac{Q}{C\rho c_p}$ (Heat source parameter)

We introduce a new variable

$$\xi = -pre^{-B\eta}, \tag{18}$$

$$\xi \theta^{"}(\xi) + [1 - pr - \xi]\theta^{'}(\xi) + \frac{p_{12}}{\xi}\theta(\xi) = 0 \tag{19}$$

Where $p_{12} = \dfrac{p_r H^*}{B^2}$

The exact solution of (16) satisfying the boundary condition (17) in the Kummer's confluent Hypergeometric function is given by

$$g(\eta) = c_1 \xi^m M(a+b;\ 1+2b;\ \xi) \tag{20}$$

Where

$$c_1 = \frac{1}{M(a+b;1+2b;-pr) - \alpha_2 B((a+b)M(a+b;1+2b;-pr) - pr(\frac{a+b}{1+2b})M(a+b+1;\ 2+2b;-pr))} \tag{21}$$

Where $a = \dfrac{pr}{2}$, $b = \dfrac{\sqrt{pr^2 - 4p_{12}}}{2}$

Case 2: prescribed surface temperature (PST)

In this case we define the non dimensional temperature and PST condition as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{22}$$

Using (4) and (22) in (2) we get, the dimensionless energy equation as

$$\theta'' + prf\theta' + pr[H^* - f']\theta = 0 \tag{23}$$

Subjected to boundary conditions

$$\theta(0) = 1 + \alpha_2 \theta'(0), \qquad \theta(\infty) = 0 \tag{24}$$

We introduce a new variable

$$\xi = -pre^{-B\eta} \tag{25}$$

$$\xi\theta''(\xi) + [1 - pr - \xi]\theta'(\xi) + [\frac{p_{12}}{\xi} + 1]\theta(\xi) = 0 \tag{26}$$

The corresponding boundary conditions are :

$$\theta = 1 + \alpha_2 \alpha_1 \frac{d\theta}{d\xi} \quad at \quad \xi$$
$$\theta = 0 \quad at \ \ \xi = 0 \tag{27}$$

The solution of (26) satisfying the boundary condition (27) in the Kummer's confluent hypergeometric function is given by

$$\theta(\eta) = c_2 \xi^m M(a+b-1;\ 1+2b;\ \xi) \tag{28}$$

Where

$$c_2 = \frac{1}{M(a+b-1;1+2b;-pr) - \alpha_2 B((a+b)M(a+b-1;1+2b;-pr) - pr(\frac{a+b-1}{1+2b})M(a+b;2+2b;-pr))}$$

4. Results and Discussion:

Here we have considered the MHD flow and heat transfer due to stretching of the sheet has been considered. The governing Partial differential equations of the flow and heat transfer are converted into ordinary differential equations by means of similarity transformation. Resulting equations of motion and heat transfer are non-linear differential equations. We has assumed exact solution of motion and using this we have obtained the solution of heat transfer analytically using the Hypergeometric series in terms of kummer's function.

We analyzed the effect of governing parameters on flow and heat transfer, illustrated in Figs.1-7.

The effect of the magnetic parameter on flow has been shown in Fig1. Which shows that as we introduce the transfer magnetic field normal to the direction of the fluid flow, due to Lorentz force, the velocity profile in the boundary layer decreases? Effect of the partial slip parameter α_1 on velocity profile is analyzed through the Fig2, It has been noticed that slip parameter has a substantial effect on the flow, on other wards, the amount of slip $1-f'(0)$ increases monotonically. The stretching of sheet does no longer impose any motion of the cooling liquid.

Fig3. Shows the effect of heat source sink, on temperature profile, It has been observed that energy is released for increasing values of H. This causes the temperature to increase, where as energy was observed for decreasing values of H which results the temperature to drop significantly near the boundary layer.

Fig4.Shows the effect of magnetic parameter on temperature profile. This plot highlights the fact that, increasing values of magnetic parameter enhance the boundary layer thickness due to Lorentz force, which produces the considerable amount of frictional heating.

Fig5. Shows the effect of velocity slip parameter 'α_1'which increases the boundary layer thickness with increasing values of α the same effect is observed in Fig6, which is plotted for analyzing effect of temperature slip parameter 'α_2' in both figures the thickening of boundary layer occurs.

Fig7. Shows the effect of Prandtle number on temperature profile, we can observe that increasing values of '$\Pr$' results in decrease of temperature distribution, which here increase of '$\Pr$' means slow rate of thermal diffusion.

Table-1:

The numerical values of wall temperature gradient.

Mn	Pr	H	α_1	α_2	$\theta'(0)$
0.5	1.0	0.05	0.1	0.1	-.0744325
			0.2		-0.69568
			0.3		-0.651897
			0.4		-0.610285
			0.5		-0.565764
0.5	1.0	0.05	0.2	0.1	-0.69568
				0.2	-0.642592
				0.3	-0.597037
				0.4	-0.557504
				0.5	-0.522885
0.5	1	0.05	0.2	0.1	-0.69568
	2				-1.12212
	3				-1.40717
	4				-1.63054
0	1.0	0.05	0.2	0.1	-1.00136
0.5					-0.69568
1					-0.231729
0.5	1.0	-0.05	0.2	0.1	-0.83433
		0			-0.773101
		0.05			-0.69568

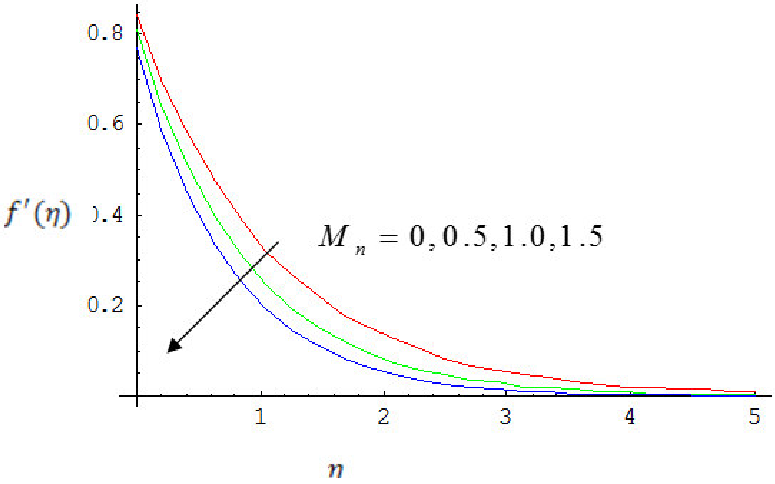

Fig1: Plot of velocity profile for various values of Mn. when other parameters is α_1 =0.2

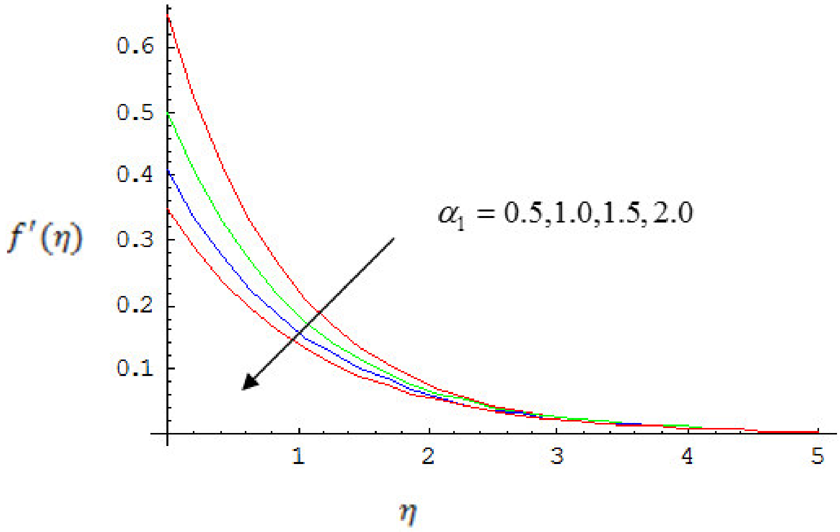

Fig2: Plot of velocity profile for various values of α_1 . when other parameters is Mn =0.5

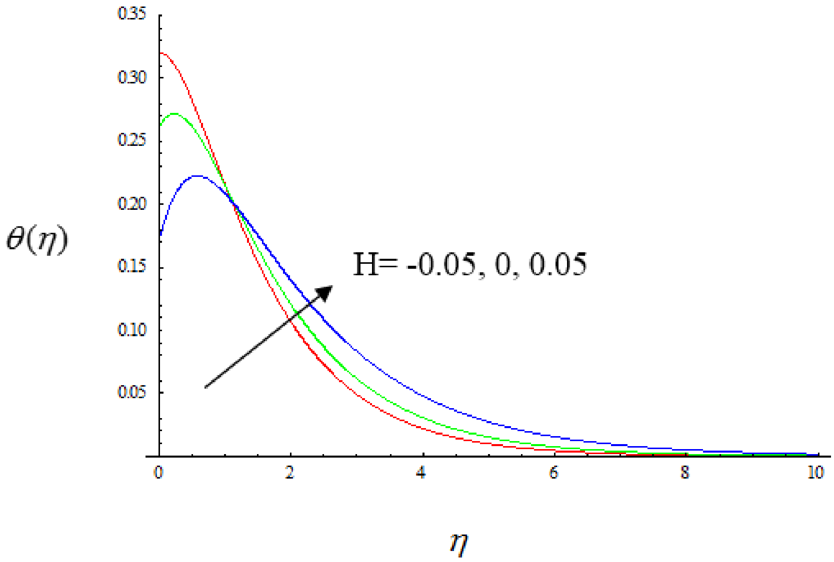

Fig3: Plot of temperature profile for various values of H. when other parameters are Pr=1.0, Mn=0.5, α_2 =0.1, and α_1 =0.2.

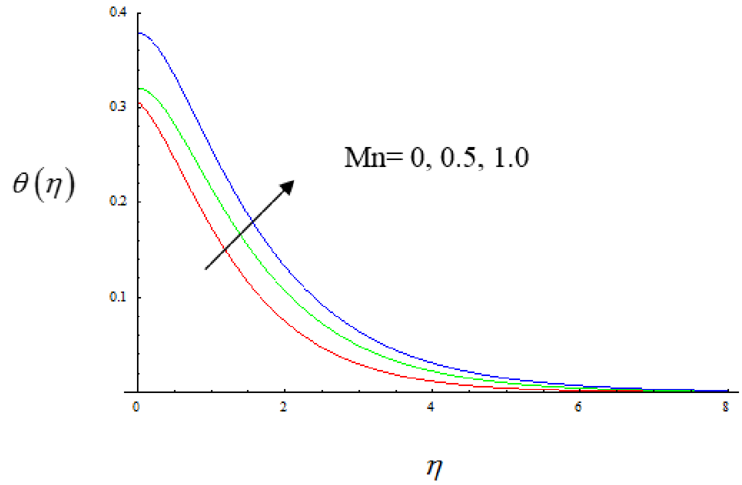

Fig4: Plot of temperature profile for various values of Mn. when other parameters are Pr=1.0, H=-0.05, α_2 =0.2 and α_1 =0.2.

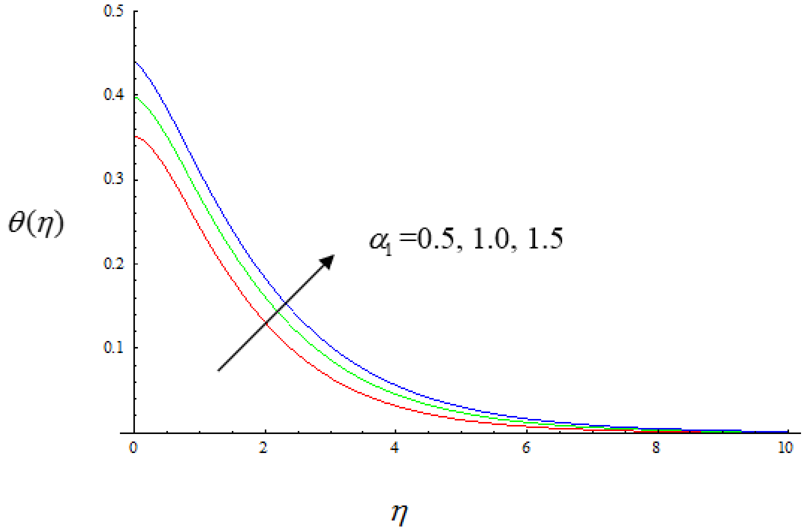

Fig5: Plot of temperature profile for various values of α_1 . when other parameters are Pr=1.0, H= -0.05 Mn=0.5, α_2 =0.2.

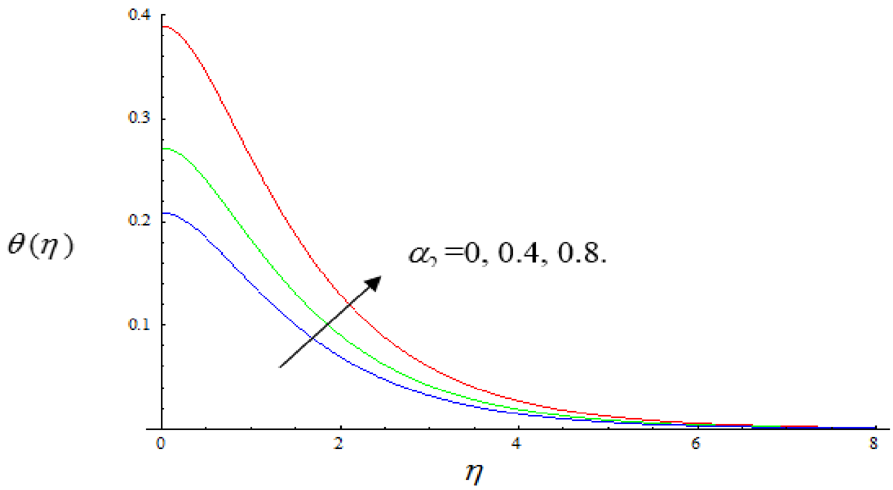

Fig6: Plot of temperature profile for various values of α_2 . when other parameters are Pr=1.0,H= -0.05 Mn=0.5, α_1 =0.2.

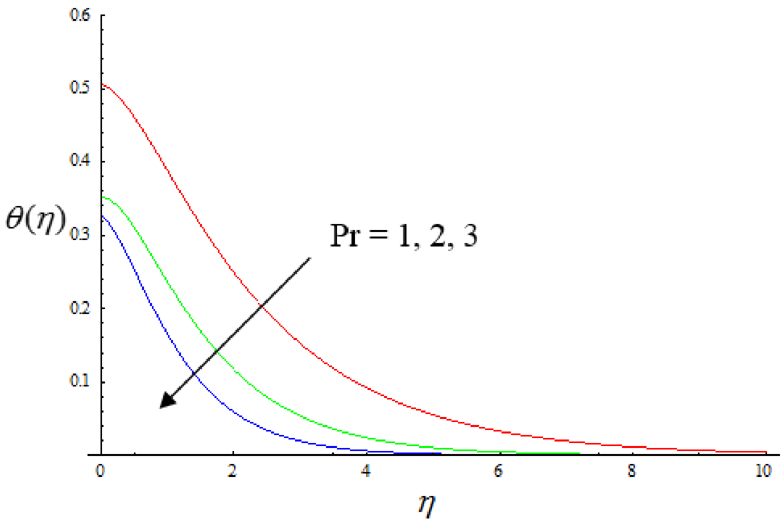

Fig7: Plot of temperature profile for various values of Pr. when other parameters are H= -0.05, Mn=0.5,n=2, α_1 =0.2, and α_2 =0.1

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